

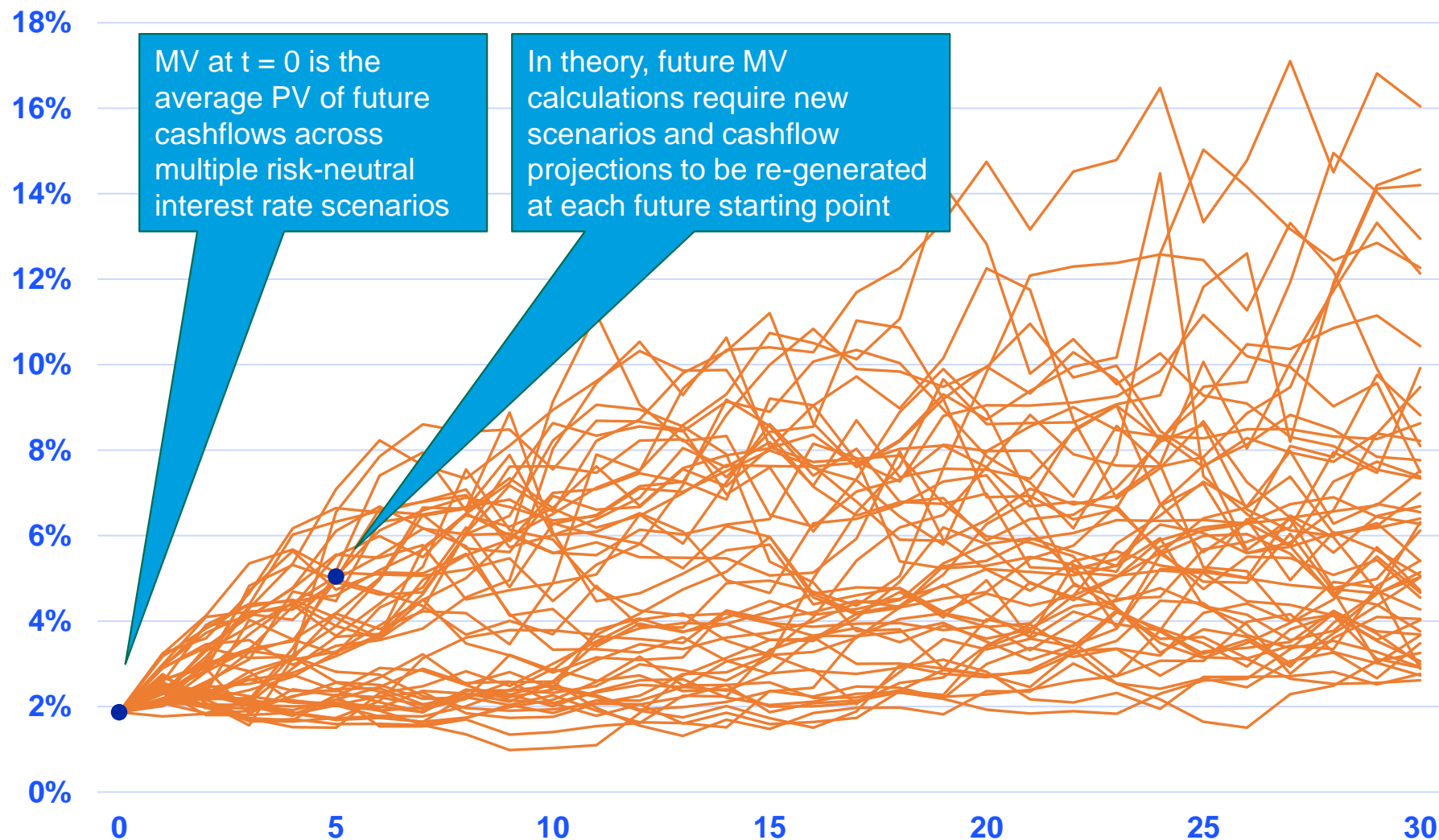
Projecting Structured Asset MVs in AXIS

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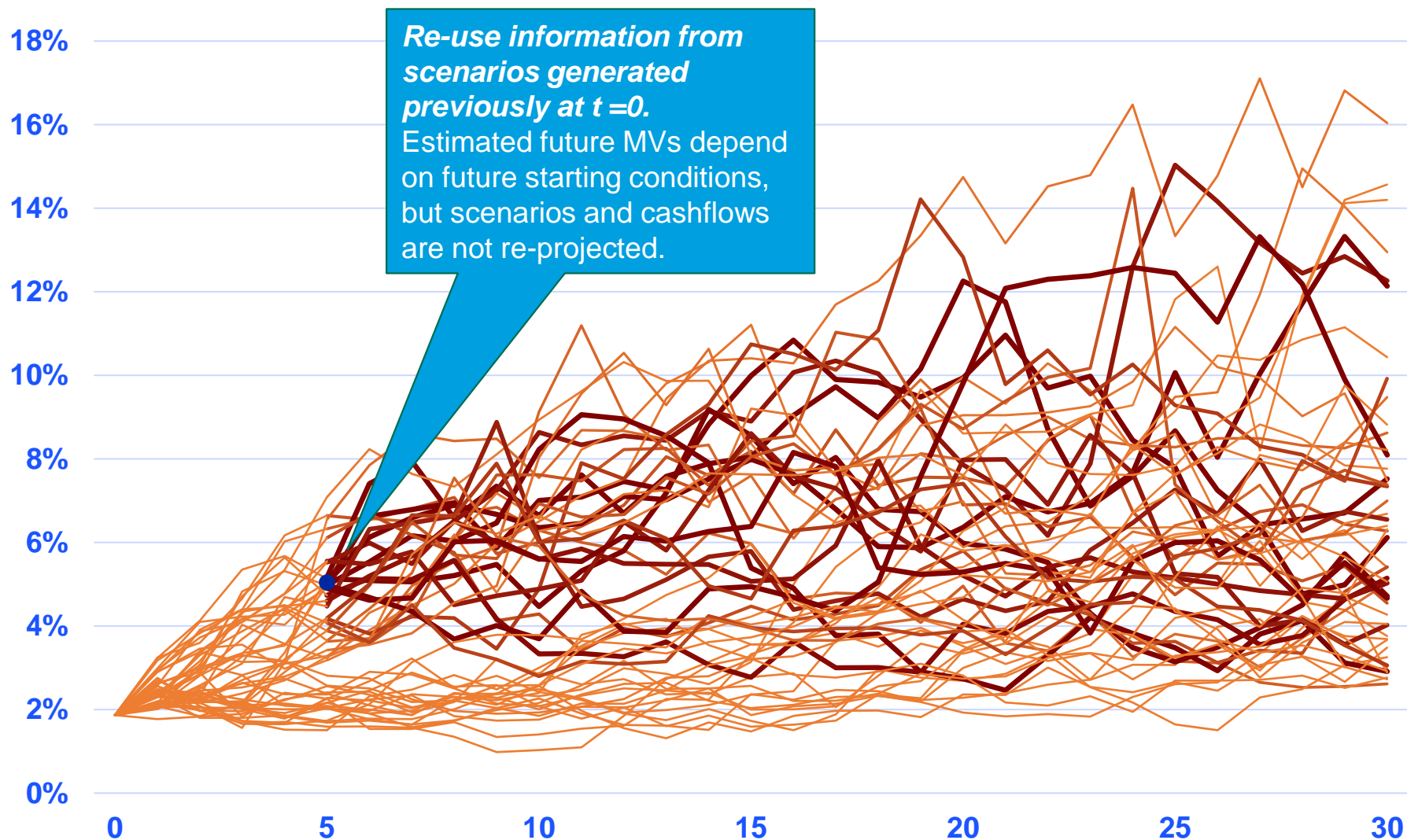
Background

- » AXIS is a multi-year projection system for life insurers' assets and liabilities
 - Projections include future cashflows, earnings and balance sheets (BV and MV)
 - Based on user-specified economic scenarios
- » AXIS is able to link to MA's Structured Finance API to project asset cashflows, but the SF API is not designed to produce future MV projections
 - SF API can calculate MVs at the projection start date, but not at future dates.
- » In theory, accurate MV projections require nested Monte Carlo simulations at each future valuation date. In practice this requires more computational resources than most users have available.
 - Currently, AXIS calculates future MVs by using the SF API to project cashflows along a single implied-forward interest rate path.
- » MA has now developed a methodology for approximating nested Monte Carlo MV calculations in AXIS with dramatically improved run-times.
 - Also provides rapid estimates of effective duration and convexity
 - Based on proxy functions, calibrated using SF API cashflow projections
 - Implementation is currently in progress

Illustrative Example: Simulated UST 10y Rates



Alternative to Nested Monte Carlo Simulation



Proxy Functions – Least Squares Monte Carlo

Objective: Estimate MV_t at some future time $t > 0$ as a function of a given vector of state variables \mathbf{x}_t (future UST rates, outstanding principal, etc.)

In general, $MV_t = \mathbb{E}^Q(PVFC_t | \mathbf{x}_t)$ where $PVFC_t$ is the present value of future cashflows as seen at time t .

Algorithm:

- » Estimate MV_0 by generating Monte Carlo simulations of $\mathbf{x}_{i,t}$ and $PVFC_{i,t}$ starting at $t = 0$. Then MV_0 is the mean of the simulated values of $PVFC_{i,0}$.
- » Assume that $MV_t \approx f(\mathbf{x}_t; \boldsymbol{\theta})$ where f is a given parametric function (a proxy function) and $\boldsymbol{\theta}$ is vector of function parameters
- » Since $MV_t = \mathbb{E}^Q(PVFC_t | \mathbf{x}_t)$, the parameters $\boldsymbol{\theta}$ can be estimated by least squares regression of the simulated values of $PVFC_{i,t}$ against $f(\mathbf{x}_{i,t}; \boldsymbol{\theta})$.
 - In effect, the simulated values of $\mathbf{x}_{i,t}$ and $PVFC_{i,t}$ provide a very large number of very poor estimates of $\mathbb{E}^Q(PVFC_t | \mathbf{x}_t)$.
 - Solve for $\boldsymbol{\theta}$ to minimize $\sum_{i,t} (PVFC_{i,t} - f(\mathbf{x}_{i,t}; \boldsymbol{\theta}))^2$
- » Well-established technique (Longstaff & Schwartz, 2001)

Model Details

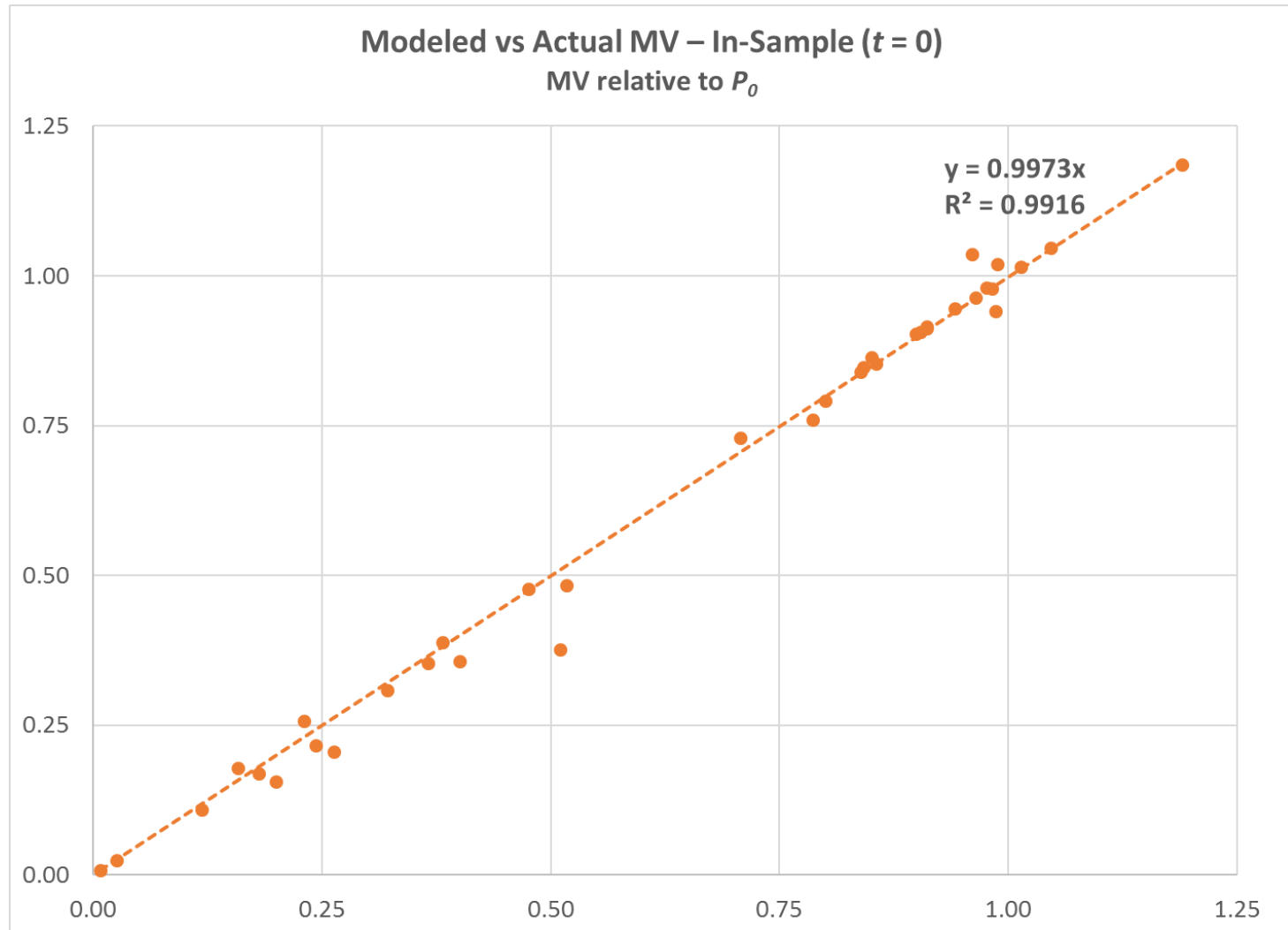
- » Assume that $g(MV_t) \approx \boldsymbol{\theta} \cdot \mathbf{x}_t$ where $g(MV_t) = \log(\exp(MV_t) - 1)$
 - Implies that $MV_t \approx g^{-1}(\boldsymbol{\theta} \cdot \mathbf{x}_t) = \log(\exp(\boldsymbol{\theta} \cdot \mathbf{x}_t) + 1)$
 - Ensures that the modeled value of MV_t is always positive even if $\boldsymbol{\theta} \cdot \mathbf{x}_t$ is negative, since $g^{-1}(y) \rightarrow y$ as $y \rightarrow +\infty$ but $g^{-1}(y) \rightarrow 0$ as $y \rightarrow -\infty$.
- » This is a **generalized linear model**
 - A generalized linear model is *not* a linear model. The link function g is non-linear, and the state vector \mathbf{x}_t can include polynomial and interaction terms.
- » Predictor variables (state variables):
 - $a_t = P_t/P_0$ where P_t is the outstanding bond principal at time t
 - $b_t = 10\text{y US Treasury rate}$
 - $c_t = 1\text{y}/10\text{y US Treasury spread}$
 - $d_t = 6\text{m USD LIBOR/Treasury spread}$
 - $\mathbf{x}_t = \{1, t, t^2, a_t, b_t, c_t, d_t, \dots, t^m a_t^n, t^m b_t^n, t^m c_t^n, t^m d_t^n\}$ where $m, n \leq 2$ (27 terms)
 - › $PVFC_{i,t}$ is discounted using Treasury rates plus a spread, but for a given bond the discount spread is constant for all i and t , and is not an explicit predictor variable
- » Parameters $\boldsymbol{\theta}$ can be constrained to match $g^{-1}(\boldsymbol{\theta} \cdot \mathbf{x}_0)$ to a given value of MV_0

Model Calibration Process

- » Tested on 44 ABS and RMBS CUSIPs from actual client portfolios
 - Selected CUSIPs all had highly variable cashflows
- » Moody's Analytics ESG 2-factor BK model used to generate 1,000 risk-neutral interest rate paths for US Treasuries and USD LIBOR
- » SAV API and Moody's Analytics MPA/PA prepayment and default models used to project cashflows and principal balances along each path
 - Projected at monthly intervals until maturity or until $P_t < 0.05 P_0$
 - Average of about 50,000 simulated values of $x_{i,t}$ and $PVFC_{i,t}$ for each CUSIP
- » Solved for θ using a standard GLM package
 - Used stepwise GLM regression to identify irrelevant predictors. Not all predictors were relevant to all CUSIPs, but each predictor was relevant to the majority of CUSIPs.
 - Also considered ordinary least squares (OLS) regression as an alternative to GLM regression. OLS is easier to implement, but results were less accurate.
 - › OLS solves for θ to minimize $\sum_{i,t} (g(PVFC_{i,t}) - \theta \cdot x_{i,t})^2$ instead of $\sum_{i,t} (PVFC_{i,t} - g^{-1}(\theta \cdot x_{i,t}))^2$.
- » Regression R^2 varied from 20.8% to 93.2%
 - A low R^2 does not necessarily imply a poor quality model. The purpose of the model is to estimate $MV_t = \mathbb{E}^Q(PVFC_t | x_t)$, not $PVFC_{i,t}$. The latter is a random variable that can have many different values for given predictor vector x_t .

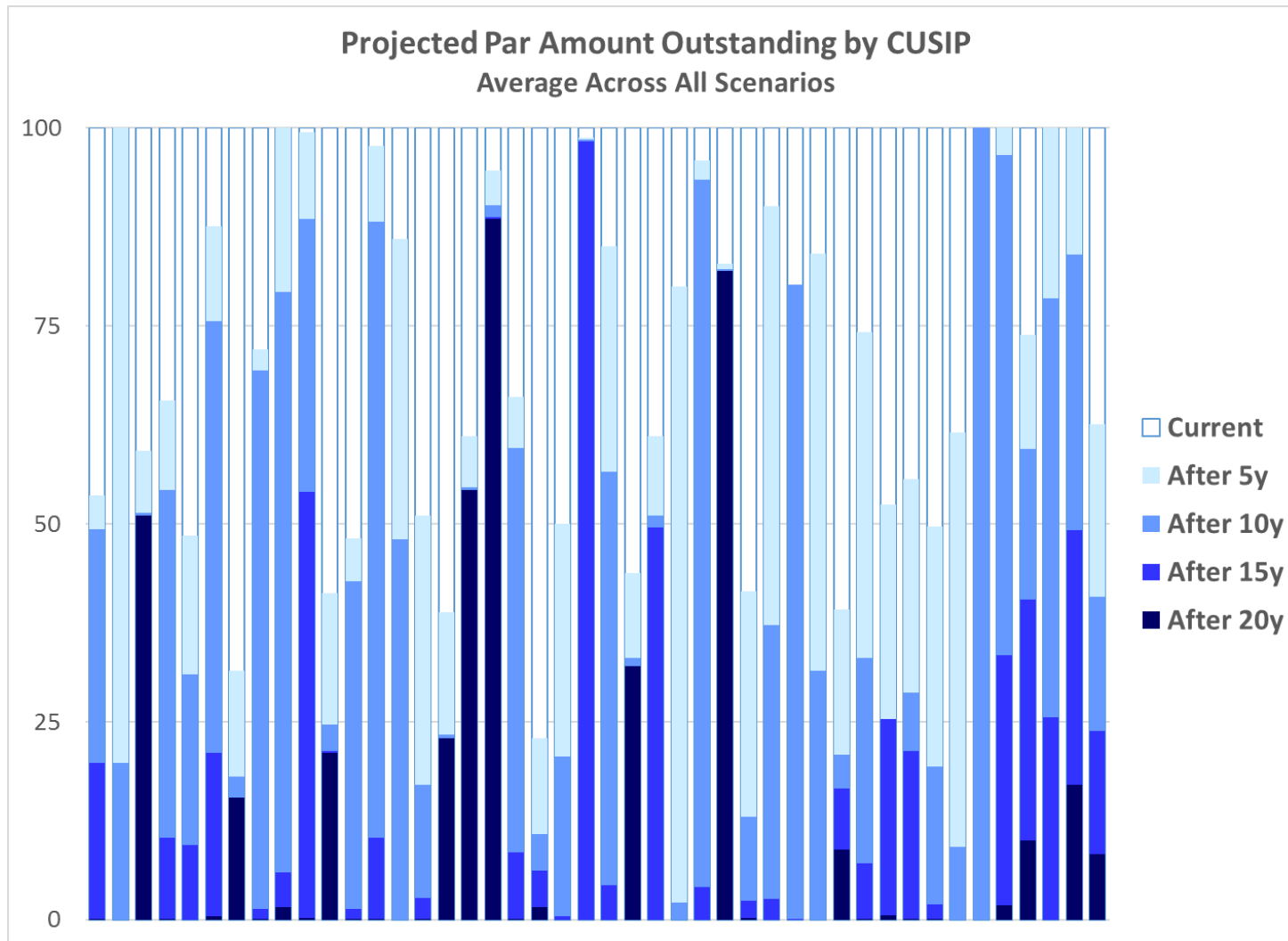
In-Sample Testing

- » Compared modeled and actual values of MV_0 (mean of $PVFC_{i,0}$) for each CUSIP



Projected Par Amounts Outstanding

- » Most CUSIPs in test sample pay down over 5-15 years

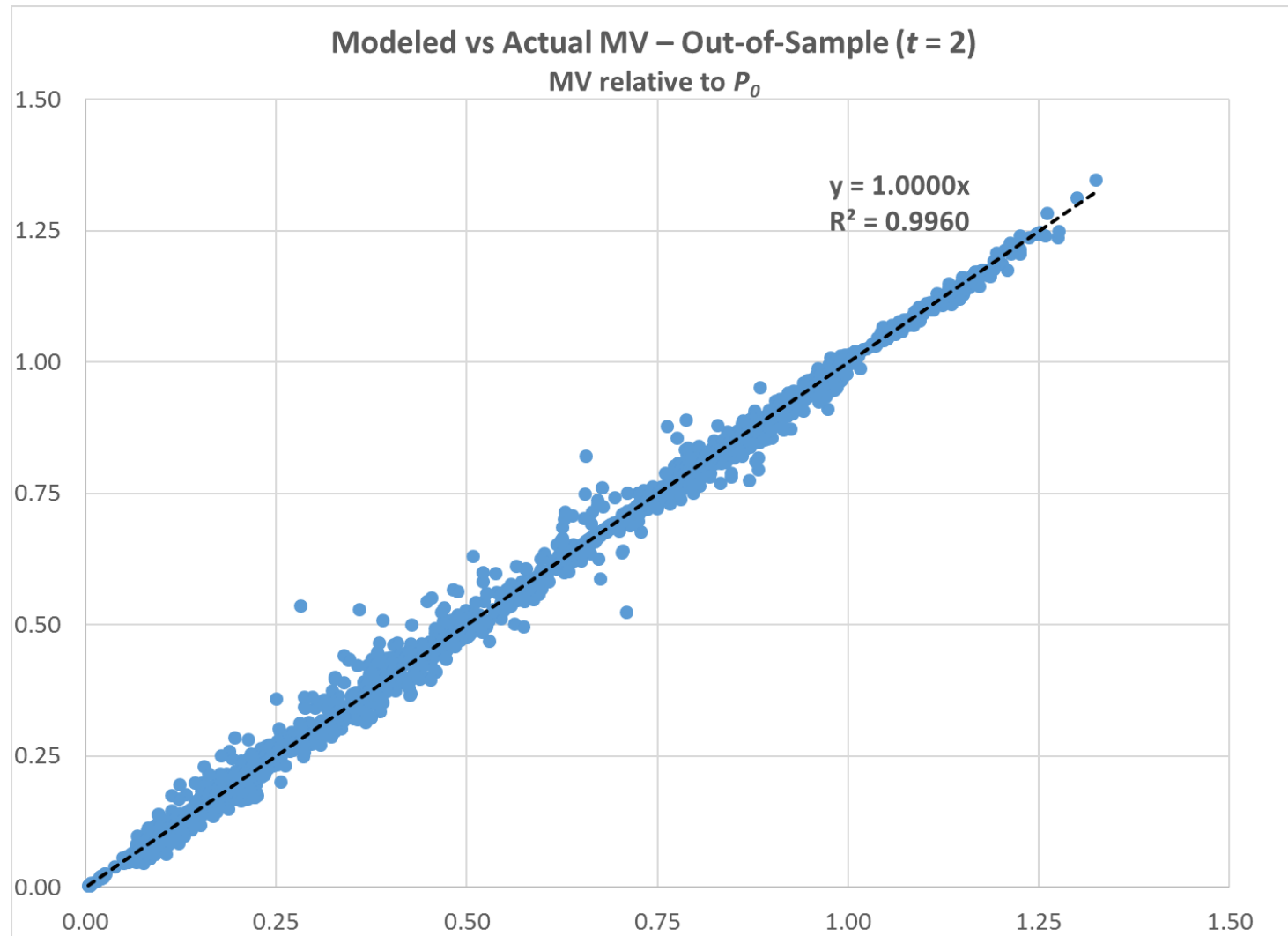


Out-of-Sample Testing

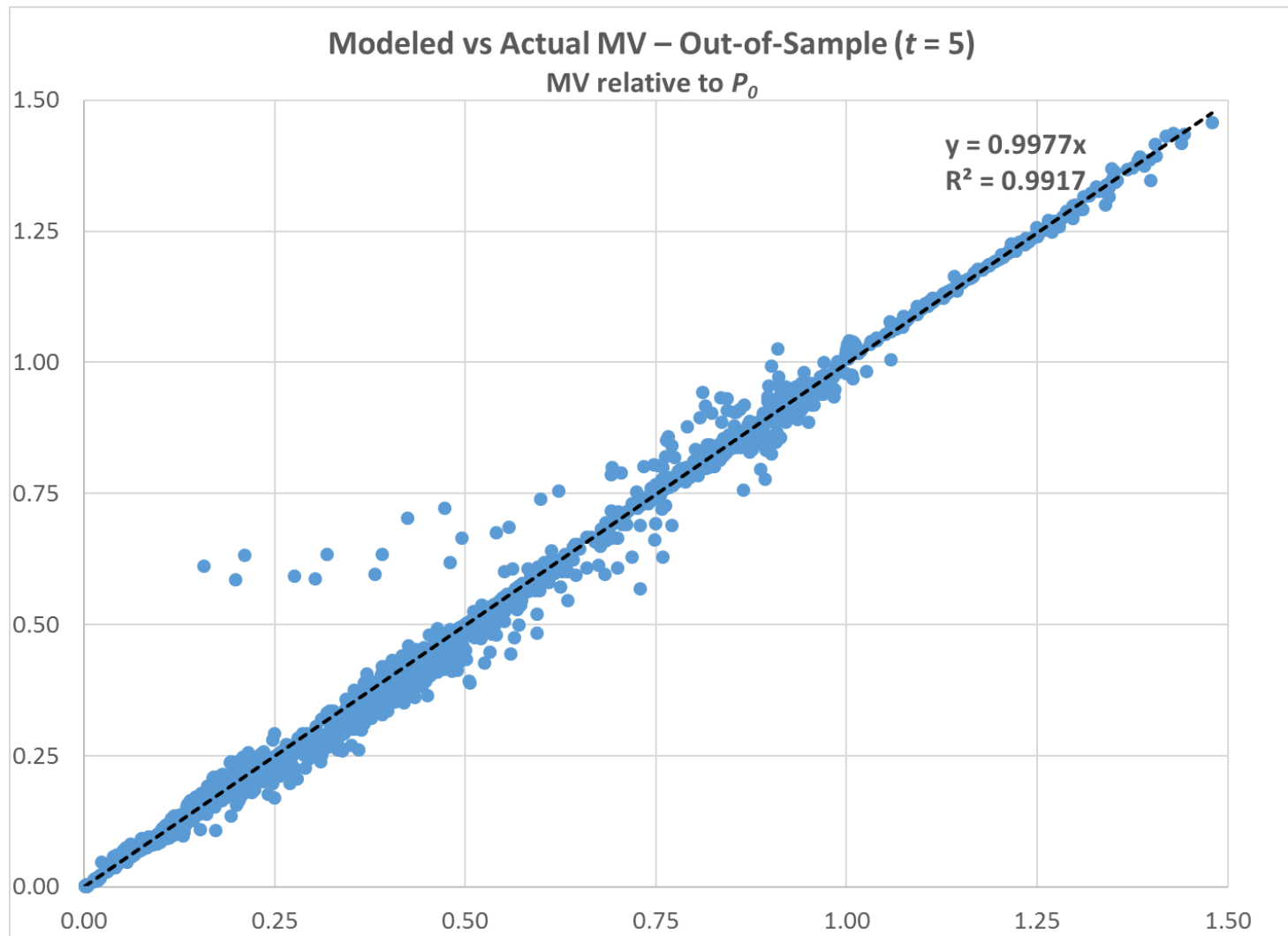
For each CUSIP, compared modeled values of MV_t at $t = 2$ and $t = 5$ to values calculated using nested Monte Carlo simulation

- » Comparisons based on 50 outer loop scenarios at each horizon, i.e. 100 outer loop scenarios in total
 - Outer loop scenarios were generated using a different interest rate model (AXIS G2++ model) than the ESG 2-factor BK model used for GLM calibration
 - Generated 1,000 G2++ scenarios and selected 50 at each horizon ($t = 2$ and $t = 5$) using stratified sampling to ensure a representative selection.
- » For each outer loop, generated 200 forward-starting risk-neutral inner loop scenarios and calculated the mean of $PVFC_{i,t}$ across inner loop scenarios
 - Inner loop scenarios were generated using the same ESG 2-factor BK model used for GLM calibration, but starting with yield curves from the outer loop scenarios
 - 880,000 calculations of $PVFC_{i,t}$ across 44 CUSIPs, 100 outer loops and 200 inner loops
- » For each outer loop, compared modeled values of $MV_t = g^{-1}(\theta \cdot x_t)$ to actual values (mean of $PVFC_{i,t}$)
 - 4,400 comparisons altogether (44 CUSIPs, 100 outer loops)

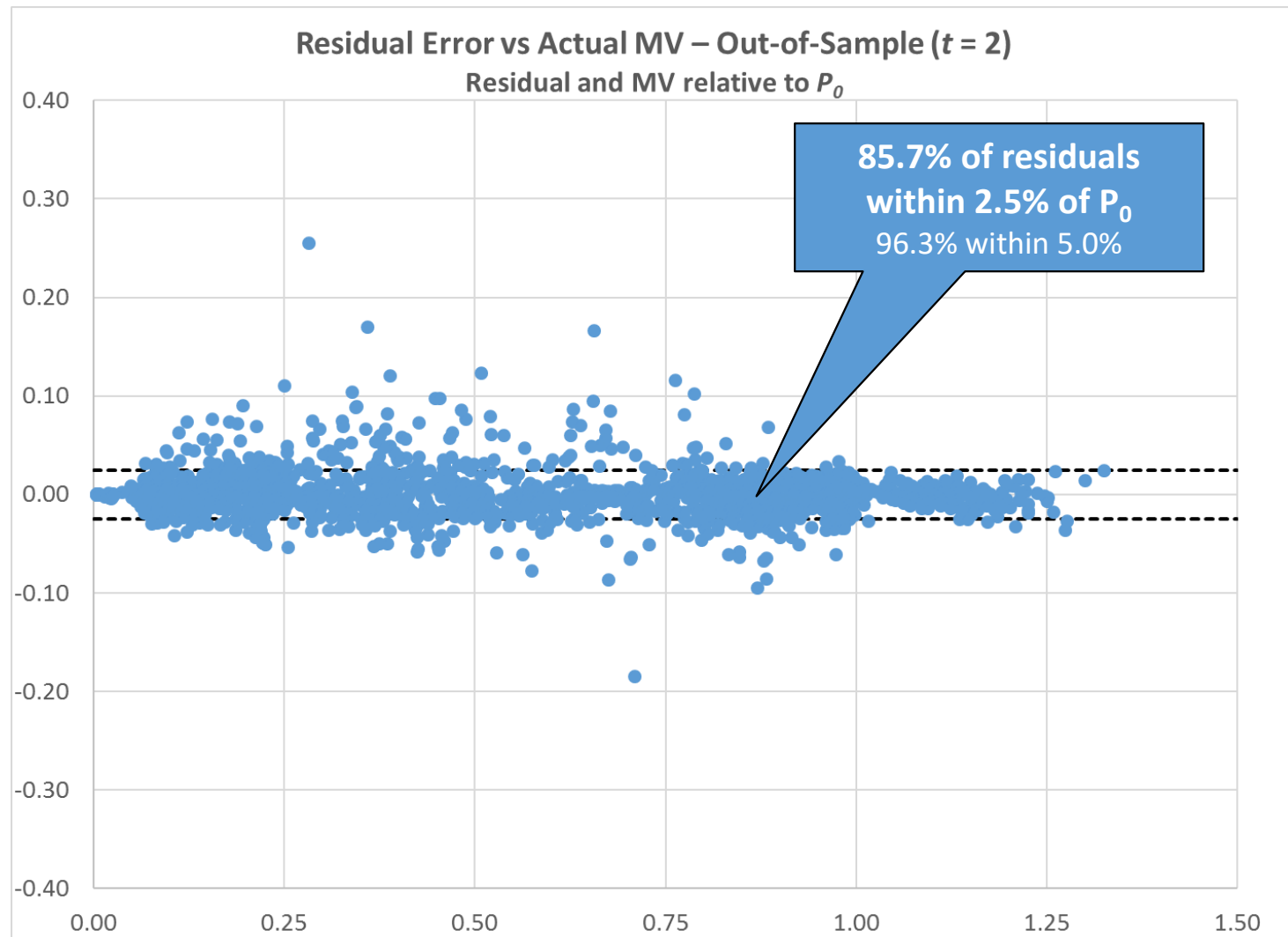
Out-of-Sample Testing ($t = 2$ years)



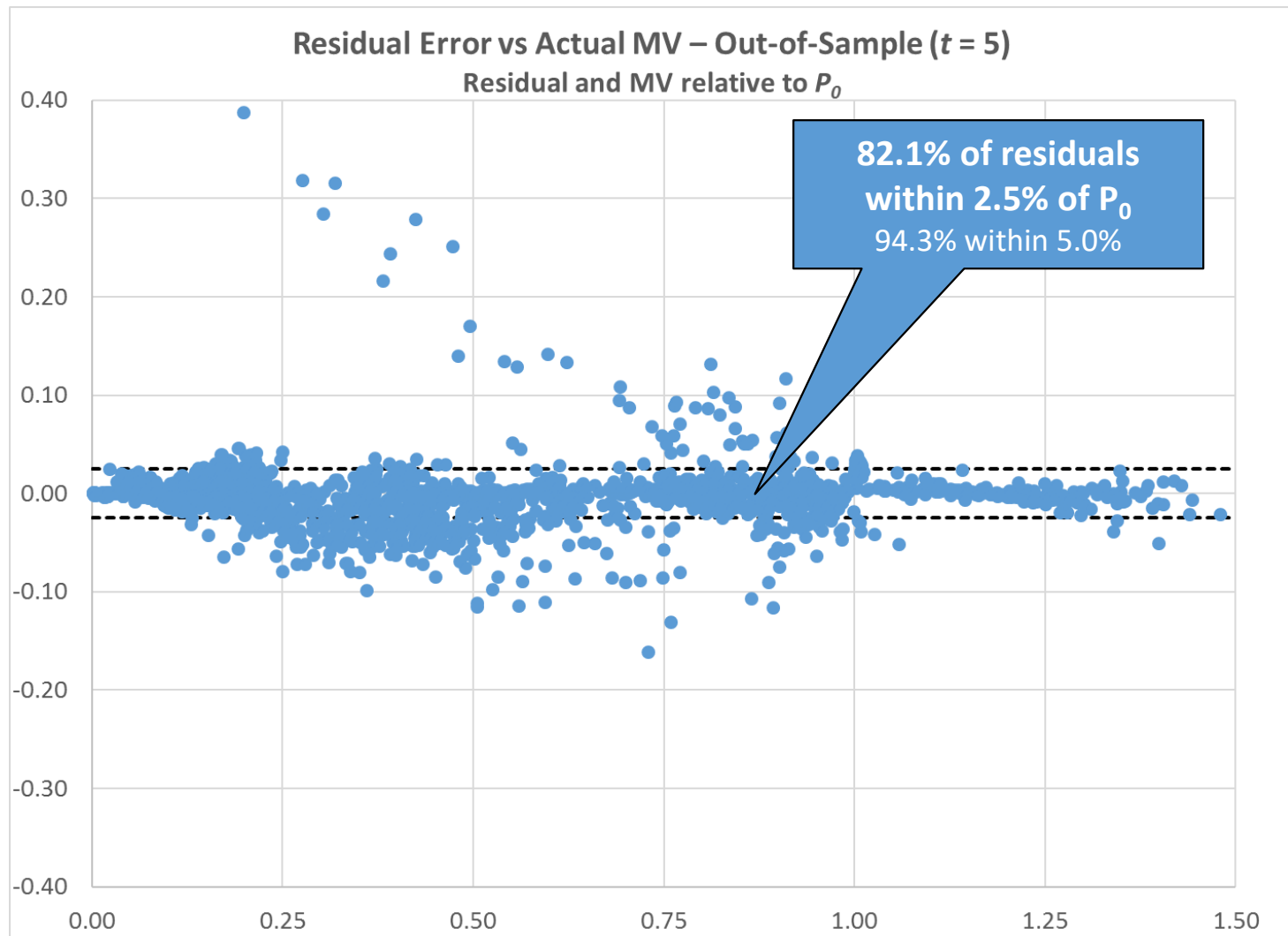
Out-of-Sample Testing ($t = 5$ years)



Out-of-Sample Residuals ($t = 2$ years)



Out-of-Sample Residuals ($t = 5$ years)



Conclusions and Observations

- » Proposed model is a very good approximation to computationally-intensive forward-starting nested Monte Carlo simulations.
 - Test results may understate the accuracy of the methodology, because the test sample consists of CUSIPs with highly variable cashflows,
- » Almost all the computational effort involves generating the initial set of scenarios and cashflow projections, i.e. $x_{i,t}$ and $PVFC_{i,t}$ starting at $t = 0$.
 - These values are available as a by-product if Monte Carlo simulation is used to calculate the initial market value MV_0
 - GLM solvers are highly efficient. Once $x_{i,t}$ and $PVFC_{i,t}$ have been generated then solving for the parameter vector θ takes only a few seconds per CUSIP.
 - Not necessary to recalibrate θ with each new production cycle. Can re-use previous calibrations.
 - Calculating out-of-sample estimates of $MV_t \approx g^{-1}(\theta \cdot x_t)$ is practically instantaneous.



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