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Multi-year projection of 1-year VaR capital requirements and free surplus

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Overview

A recent research report presented methodologies and case studies for the development of proxy functions for use in efficient multi-year projection of the market-consistent liability values of complex life liabilities. These approaches were shown to be similarly applicable to the multi-year projection of run-off reserves such as those used in North American statutory capital. This report further extends the applicability of these methodologies to a third application: the multi-year projection of 1-year VaR capital requirements. Such a capability will allow firms to reliably project the behavior of their free surplus and hence their potential future capital needs. It could therefore be of significant importance for the quantitative modeling elements of ORSA. As in earlier reports, the technical methodology is developed and then investigated through the use of a case study.

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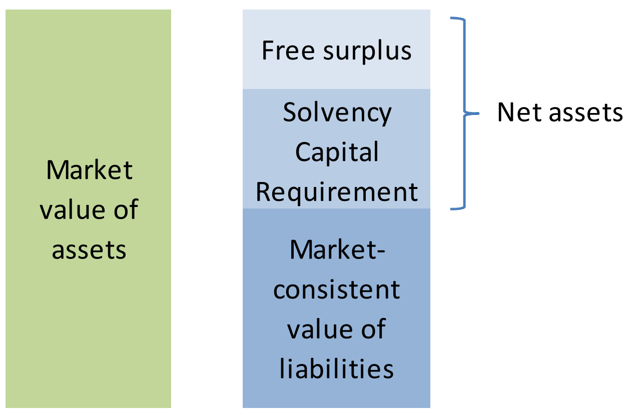
1. Introduction

Emerging global regulatory requirements such as Own Risk and Solvency Assessment (ORSA), along with general business planning requirements, have resulted in the requirement for insurers to understand how their balance sheet and solvency position progresses over time across a range of scenarios[1]. In a previous report (Morrison, Turnbull and Vysniauskas, Multi-year projection of run-off conditional tail expectation (CTE) reserves 2013) we considered the projection of reserves and capital requirements based on a CTE measure of the type that is popular in North America. In this note we consider the projection of a Solvency II-style balance sheet, based on market-consistent valuation of assets and liabilities and a 1-year 99.5% VaR measure of solvency capital, as shown in Figure 1. Under Solvency II's ORSA, and similar business planning exercises, each of these items may need to be calculated many times – under a number of different scenarios, over a number of time-steps.

Figure 1: Illustrative Solvency II balance sheet

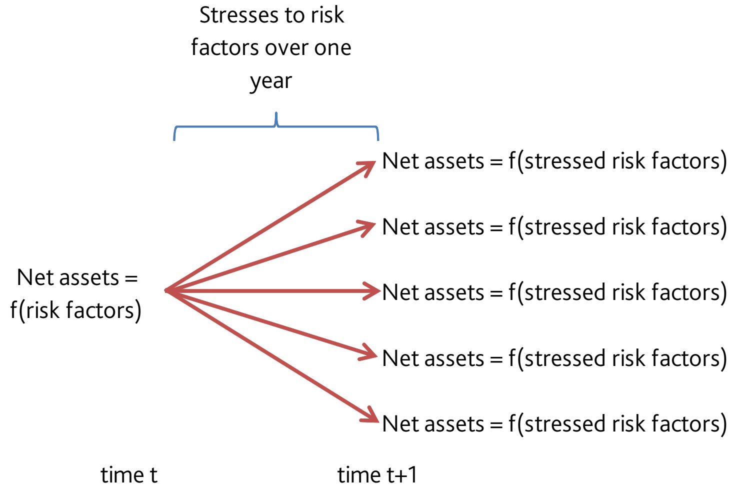


Evaluating this balance sheet for a typical insurance firm, even at a single point in time in a single scenario, typically involves a huge amount of computational effort. While the market value of assets is usually straightforward to observe or calculate, calculation of the market-consistent value of liabilities generally requires risk-neutral Monte Carlo simulation, which is computationally costly. In a previous paper (Morrison, Turnbull and Vysniauskas 2013) we described a methodology for developing *multi-step proxy functions* - formulae which approximate the market-consistent value of liabilities as a function of risk factors, allowing these liability values to be calculated quickly within a multi-step projection exercise without having to resort to risk-neutral simulation. Armed with these proxy functions, we demonstrated how the market-consistent value of liabilities (and hence net assets) can be projected over time under a range of stochastic and deterministic scenarios that might be typically used as part of a firm's ORSA.

However, knowledge of future asset and liability values alone doesn't give a full picture of the future solvency position of the firm. For that we additionally need to calculate the future Solvency Capital Requirements (SCRs). Suppose we want to calculate the SCR in some scenario, at some future time t. In Solvency 2, the SCR is defined as a 1-year Value-at-Risk, which can be estimated using a simulation approach (typical of Internal Models) or a formula (as in the Standard Formula) or a combination of the two. Though the details of the calculation will differ depending on the approach adopted, in all cases the SCR is estimated by projecting risk factors forward by one year (to time t+1) under a range of risk factor stresses, and evaluating the change in net assets in each stress, as shown in Figure 2.

[1] For further background to the requirements of ORSA, see (Turnbull and Frepp 2013).

Figure 2: Estimation of SCR, at future time t, using proxy functions



In the case of an Internal Model, the stresses to risk factors represent one year real-world stochastic scenarios, while in the Standard Formula the stresses are prescribed. In both cases, net assets (or Own Funds, in SII parlance) need to be evaluated in each of the stress scenarios, and again the valuation of liabilities in this calculation presents a computational challenge in principle. However, the availability of multi-step proxy functions allows us to carry out each valuation analytically without resorting to risk-neutral simulation. Thus the SCR, and the entire balance sheet, can be evaluated relatively quickly within an ORSA projection, either using an Internal Model or Standard Formula approach.

In this paper, we illustrate the technique using a Standard Formula approach to estimate the SCR (see Appendix), though the methodology extends in an obvious way to the Internal Model approach.

2. Example liability and proxy function

The application of proxy functions to multi-year projection of an insurance company's balance sheet will be illustrated using a case study. We will consider sample policy with payout linked to the performance of a corporate bond portfolio, with a guarantee applied annually.

The main liability assumptions are summarized below:

» Assume an annual return of max (fund return – 1.5%, 2%) is credited to the policy account,

i.e. Policy Account(t) = Policy Account(t-1) x (1 + max (Fund Return(t-1,t) – 1.5%, 2%))

and Policy Account(0) = Fund Value(0)

» The underlying fund is a diversified portfolio of US corporate bonds. The bonds are assumed to be invested with a credit mix of 70% A-rated and 30% BBB-rated, and with a term of 8 years. The bonds' credit rating and term are assumed to be re-balanced annually.

» The policyholder is assumed to exit the policy after ten years, and will receive the value of the policy account at that point.

» No allowance is made for tax, mortality, expenses or lapses.

We note that the payout on this policy is highly path-dependent. The annual return credited to the policyholder has a year-on-year guarantee of 2%, and so the payout at year ten depends not just on the final value of the underlying investment fund, but on each annual return over the 10-year period. While this path dependency would normally mean that risk-neutral Monte Carlo simulation is required for valuation, (Morrison, Turnbull and Vysniauskas 2013) developed multi-year proxy functions for the liability value as an alternative to simulation, and demonstrated that these proxy functions provide an accurate estimate of the actual market-consistent liability value in a wide range of future scenarios, including scenarios which are considered particularly adverse to the shareholder, in the sense that the market-consistent value of liabilities significantly exceeds the value of the underlying bond fund. These scenarios correspond to extreme values of risk factors, in this case high levels of risk-free interest rates and/or credit spreads. The assessment of the balance sheet under such adverse scenarios is likely to be an important part of any ORSA exercise.

However, in the estimation of Solvency Capital Requirements, we may need to apply the proxy function under scenarios that are even more extreme than those considered in the 'base case' valuation. As shown in Figure 2, estimation of the SCR requires evaluation of the proxy function under *stressed* values for the risk factors at time t+1. Thus the proxy function needs to be evaluated at ultra-extreme values of the risk factor space.

For this reason, we have chosen fitting scenarios which span a wider range than in our previous paper. Furthermore, we have generated these fitting scenarios using a real-world scenario generator, with the 2FBK interest-rate model being used to generate fitting scenarios for risk-free interest rates. In order to generate fitting scenarios in the extreme tails of the risk factor space, we used a combination of two methods:

(1) Supplementing the base real-world calibration with a number of stress calibrations, corresponding to stresses to the initial value of credit spreads and/or addition of (negative) term premia so as to generate high interest rate scenarios. This ensures that we generate more adverse scenarios than we would using a base calibration alone.

(2) Stressing the resulting scenarios by applying the Standard Formula stresses. This creates the type of ultra-extreme scenarios required to estimate the SCR.

Note that the use of the 2FBK model to generate fitting scenarios contrasts with our previous paper where we used a market consistent scenario generator, with the LMM+ interest rate model being used to generate fitting scenarios for risk-free rates. Using the LMM+ model, we find that we cannot generate an appropriate range of fitting scenarios for the purpose of projecting the SCR, even under stressed calibrations – differences in the structural properties of the 2FBK and LMM+ models mean that certain shapes of yield curve that are produced by 2FBK are not generated at all using LMM+.

A potential drawback of using the 2FBK model to generate fitting scenarios is that this requires more computational effort than required using LMM+. As discussed in our previous paper, the use of the same (LMM+) model to generate fitting points *and* to generate the risk-neutral scenarios used for valuation means that the same scenarios can be used to fit functions at different projection times, so that the computational requirements are independent of the number of such times. By using a different model to generate fitting scenarios from that used to generate risk-neutral valuation scenarios, as we have done here, the computational time required scales linearly with the number of fitting times. As is often the case, there is a tradeoff between accuracy and computational time, and here we have chosen to accept a larger computational burden in order to achieve a higher accuracy.

Using the Least Squares Monte Carlo technique, we calibrated a 'global' proxy function depending on time plus four risk factors[2]:

» Yield curve 'level' (the one year risk-free spot rate)
» Yield curve 'slope' (the ten year risk-free spot rate minus the one year risk-free spot rate)
» 10-year BBB credit spread
» The credited policy account at the time of valuation

Fitting points were generated a times 1, 5 and 9 and the B&H Proxy Generator used to perform the fit. The fitted proxy function is a polynomial with 15 terms, with the maximum order of any individual term (including cross terms) being two.

3. Capital management assumptions

In order to simplify the description, we summarize the various items in the balance sheet in shorthand notation, as follows:

» A(t) = Market value of assets at time t
» L(t) = Market-consistent value of liabilities at time t
» SCR(t) = Solvency Capital Requirement at time t
» FS(t) := A(t) – [L(t) + SCR(t)] = Free surplus at time t

Initially (at t=0), we assume that we hold total assets such that the free surplus is FS(0) = 0.1 x [L(0) + SCR(0)], i.e. A(0) = 1.1 x [L(0) + SCR(0)], with 1 being invested in the bond portfolio on which the guarantee is written, and the rest being invested in cash.

[2] In our previous paper, we also included a variable representing the level of interest-rate volatility. However, since we are not using a stochastic volatility interest rate model to generate fitting scenarios here, this variable is no longer required.

At t=0, we estimate the initial market-consistent value of liabilities to be 1.189, and the SCR to be 0.231. Therefore initial total assets are 1.562 (and starting free surplus is 0.142), with 1 invested in the bond fund and 0.56 invested in cash[3].

At any future point in time t, having calculated the market-value of assets A(t)[4], market-consistent value of liabilities L(t) and Solvency Capital Requirement SCR(t) using the fitted proxy functions, we calculate the free surplus FS(t) = A(t) – [L(t) + SCR(t)]. We then assume that capital is injected, or dividend payments made, in order to achieve a target level of free surplus as determined by the following rules:

- In the event of insolvency, FS(t) < 0, inject capital to bring free surplus up to FS(t) = 10% x [L(t) + SCR(t)]
- If FS(t) > 10% x [L(t) + SCR(t)], pay dividend, reducing free surplus to FS(t) = 10% x [L(t) + SCR(t)]
- If 0 <= FS(t) <= 10% x [L(t) + SCR(t)], neither inject capital nor pay dividend

Any capital injections are invested in cash, and dividends are paid out of existing cash investments.

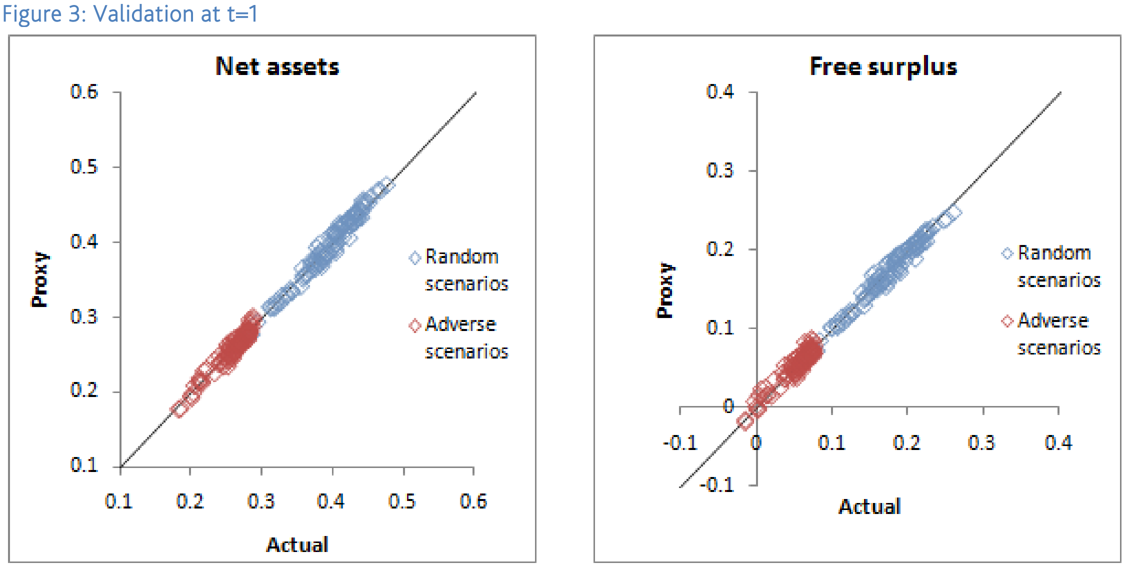
4. Validation of the proxy function

Having fitted the proxy function, we validate it under a selection of out-of-sample scenarios for the risk factors. These scenarios were selected as follows.

Firstly, 10,000 real-world scenarios were generated using the Barrie & Hibbert Economic Scenario Generator, calibrated at end-December 2012. A validation of the proxy function in all 100,000 real-world scenarios would be a computationally costly exercise, requiring full nested simulation. Our approach to validation involves selecting a subset of representative 200 scenarios consisting of 100 'random' scenarios, and an additional 100 'adverse' scenarios, in which the estimated market-consistent value of the policy significantly exceeds the value of the underlying fund. For each of these 200 validation scenarios, the 'actual' market-consistent value of the policy, and SCR, were estimated using 5,000 market-consistent scenarios. From this we calculated net assets and free surplus (prior to any dividends/capital injections) and compared these with the corresponding quantities calculated using the proxy function[5].

Figures 3-7 show this comparison at years 1, 3, 5, 7 and 9.

Figure 3: Validation at t=1



[3] Total assets(t=0) = 1.1*(1.189+0.231)=1.562

[4] Assets were valued exactly using simulated yield curve and spread scenarios.

[5] Note that net assets here includes cash assets, calculated using the management assumptions set out in Section 3. Due to these management assumptions, the level of cash assets at any future point in time depends on the path of the liability value and SCR up to that point. In order to minimise the number of 'actual' valuations we have estimated this path (of previous liability values and SCRs) using proxy functions, and only ever estimated 'actual' values at the point of valuation.

Figure 4: Validation at t=3

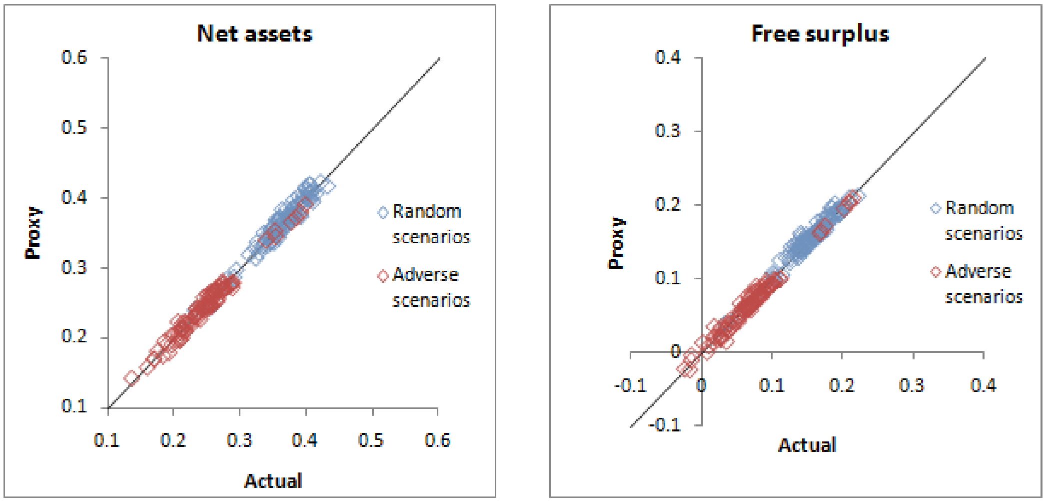


Figure 5: Validation at t=5

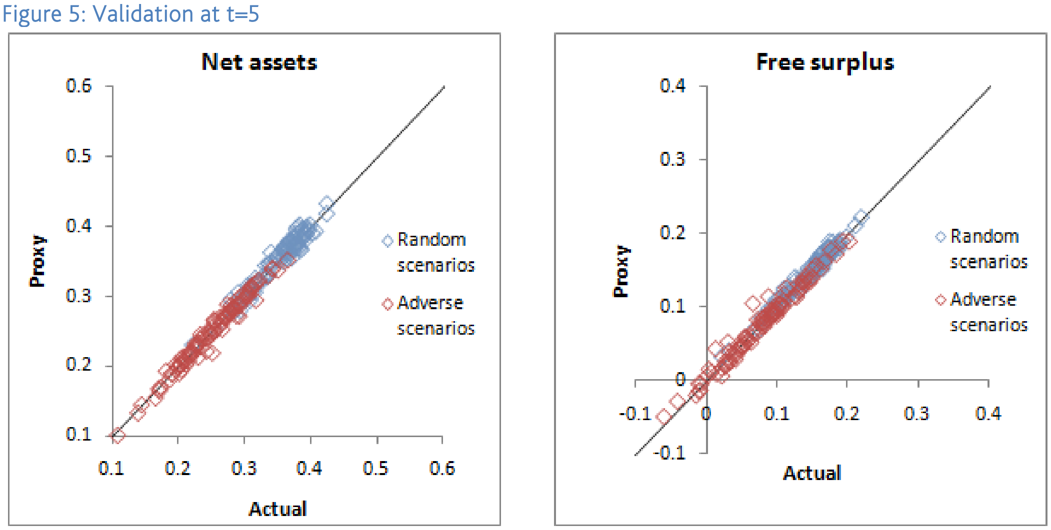


Figure 6: Validation at t=7

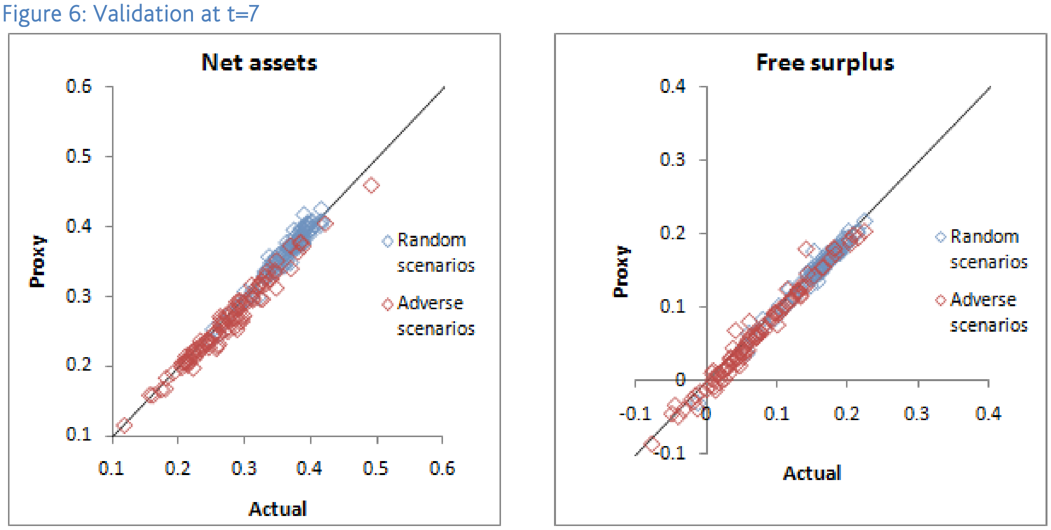
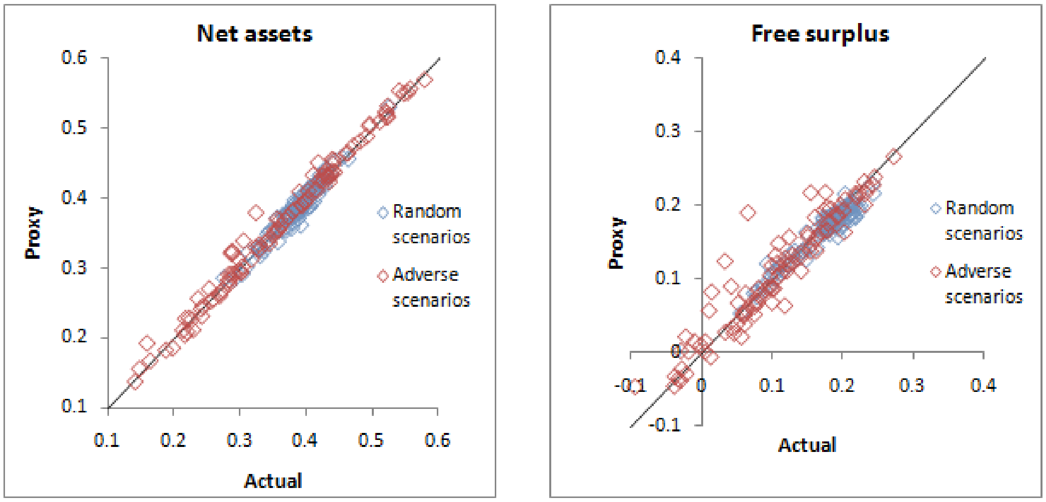


Figure 7: Validation at t=9



We observe that in general a good fit is achieved for both net assets and free surplus, in both random and adverse scenarios. The largest discrepancies between the proxy and actual values occur in the adverse scenarios, as expected given that these correspond to the extreme tails of the risk factor space, which are less represented in the set of fitting scenarios. The quality of fit also appears to deteriorate somewhat as we extend the projection horizon, particularly in the case of free surplus. It should be noted that for the fit shown here, a smaller number of fitting scenarios were placed at year 9, and it may be possible to improve the fit at later years by placing more fitting scenarios at year 9 (possibly at the expense of the quality of fit at earlier times) or by performing 'local' fits.

5. Projecting the business's future capital requirements and shareholder cash flows

In this section we use the proxy method developed above to project the behavior of the Standard Formula SCR and the business's free surplus across thousands of real-world 10-year stochastic scenarios. Figures 8 and 9 show the probability distributions generated for the Standard Formula SCR in monetary units and as a proportion of the market-consistent liability value respectively.

Figure 8: Real-world 10-year projection of Standard Formula SCR

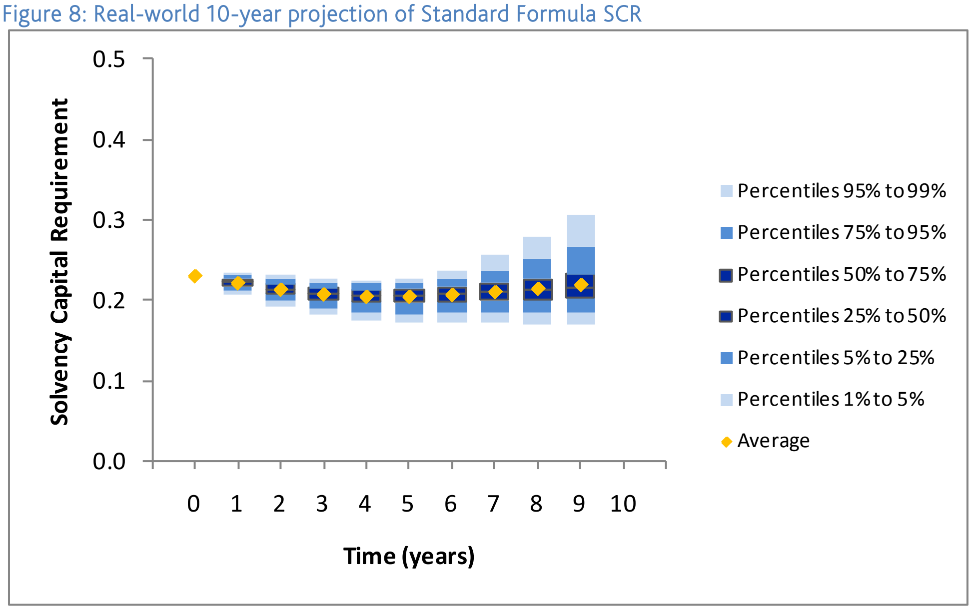


Figure 9: Real-world 10-year projection of Standard Formula SCR as a % of market-consistent liability value

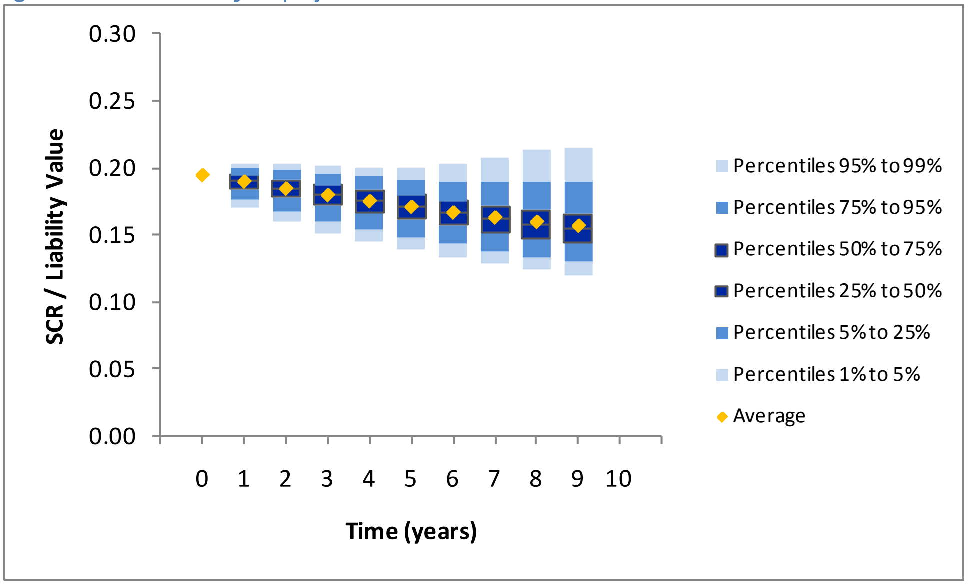
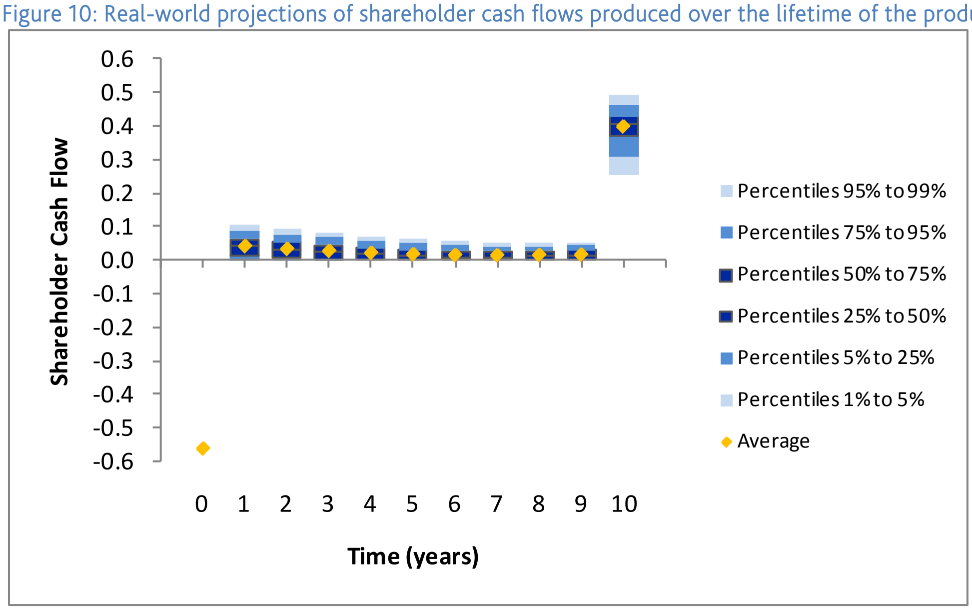


Figure 9 highlights the potential limitation of using a carrier method that assumes SCR is a constant proportion of the projected market-consistent liability value – in our case study product, the year 9 SCR could be anywhere between 13% and 22% of the market-consistent liability value, and this could often be the difference between being solvent or insolvent.

We now consider what the above behavior of SCR and free surplus implies for shareholder cash flows – both capital injections and dividend payments. You may recall from section 3 above that we assume the business is initially capitalized such that the starting free surplus is equal to 10% of the sum of the market-consistent liability value (1.189) and Standard Formula SCR (0.231). This implies starting free surplus of 0.142, which in turn requires total starting assets of 1.562. As the policyholder pays a premium of 1 to buy the product, the shareholder therefore must provide capital of 0.562 at the inception of the policy. Figure 10 shows the probability distribution for the shareholder cash flows that occur following this initial investment.

Figure 10: Real-world projections of shareholder cash flows produced over the lifetime of the product



Dividends are paid each year when the free surplus exceeds the 10% margin, and further capital injections are paid when free surplus falls below zero. Figure 10 shows that, in our projections, it is very rare for the 10% margin to be completely exhausted, and a small dividend payment is possible in most years. A large dividend is usually payable at the maturity of the policy as the SCR capital is released back to the shareholder. Figure 11 shows the probability distribution for the internal rate of return generated by these 10-year shareholder cash flow streams.

Figure 11: Internal rate of return on shareholder cash flows

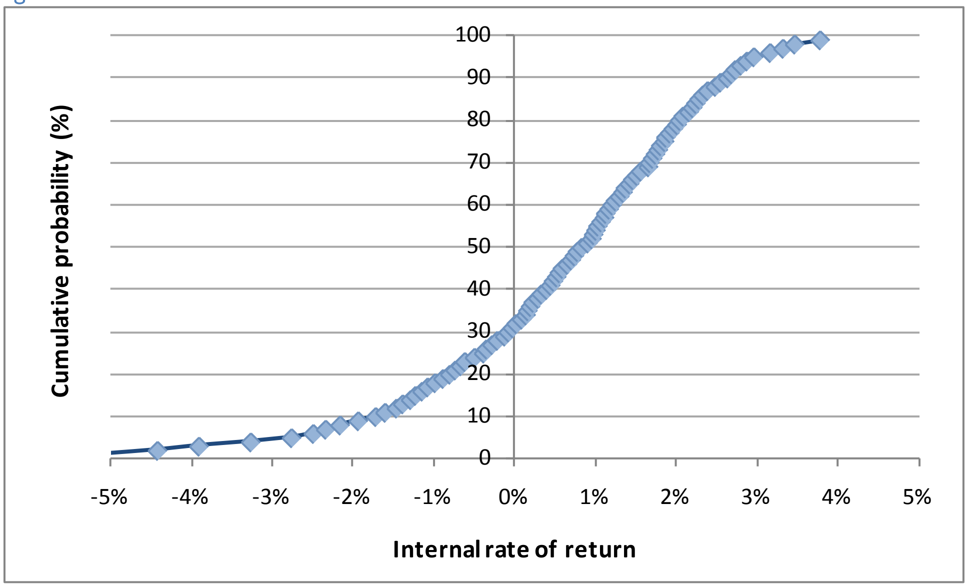


Figure 11 shows that the average shareholder IRR from writing this product is very low – the median is around 1% per annum. This result is driven by two key reasons: first, the cash provided by the shareholder is assumed to be invested in cash throughout the term of the policy; and secondly, the product that is being underwritten by the shareholder is not particularly profitable (indeed, on a market-consistent basis, the product guarantee makes it highly unprofitable).

6. Conclusions

This paper has extended our earlier published research in multi-period proxy modeling by developing methodologies for the projection of 1-year VaR capital requirements as well as market-consistent liability values. We again find that Least Square Monte Carlo-based fitting methods can provide an efficient and accurate means of making multi-year projections of complex liability and capital metrics.

The projection of VaR capital can be particularly useful in the context of analyzing future capital adequacy for purposes such as ORSA. The case study developed in this paper highlighted overly simplistic methods that assume future Solvency Capital Requirements will be stable constant proportions of asset or liability values may lead to misleading conclusions about capital adequacy, especially for complex liabilities with guarantees.

Appendix: SCR calculation using the standard formula

In this paper we assume that the SCR is calculated using the Standard Formula. This Appendix describes the calculation, based on the EIOPA technical specification on LGTA (Eurpoean Insurance and Occupational Pensions Authority 2013).

Given market value of assets A(t) and market-consistent value of liabilities L(t), the Net Asset Value NAV(t) is defined:

$$NAV(t) = A(t) - L(t)$$

Based on the EOPIA technical specification, the Standard Formula approach to calculating the SCR involves performing t=0 stresses to individual risk drivers, obtaining changes in Net Asset Value positions under the stress scenarios, and combining the resulting NAV changes using a correlation based formula. For this particular case study the stressed risk drivers are interest rates and credit spreads. Therefore, the NAV value under stress scenarios can be determined using the following:

$$NAV_{Stressed}(t) = A_{Stressed}(t) - L_{Stressed}(t)$$

Where the *stressed* subscripts identify up/down interest rate and credit spread stresses as defined in the EOPIA technical specification. The change in the NAV(t) for the interest rate stress is defined as a combination of specified upward and downward stresses to risk-free interest rates:

$$\Delta NAV_{IR\ Stress}(t) = Min\{NAV_{IR\ Stress\ Down}(t) - NAV(t), NAV_{IR\ Stress\ Up}(t) - NAV(t), 0\}$$

The change in NAV(t) for the credit spread stress is defined as:

$$\Delta NAV_{Credit\ Spread\ Stress}(t) = Min\{NAV_{Credit\ Spread\ Stress}(t) - NAV(t), 0\}$$

The overall SCR at a given point in time and current state of risk drivers (i.e. current yield curve level and credit spreads) is defined by correlating the changes in Net-Asset-Values under stresses to different risk factors:

$$SCR(t) = \sqrt{\Delta NAV(t) \sum \Delta NAV(t)^T}$$

where $\Delta NAV(t)$ is a vector of changes in NAV:

$$\Delta NAV(t) = [\Delta NAV_{IR\ Stress}(t), \quad \Delta NAV_{Credit\ Spread\ Stress}(t)]$$

and $\sum$is a correlation matrix[6]:

$$\Sigma = \begin{bmatrix} 1 & 0.5/0 \\ 0.5/0 & 1 \end{bmatrix}$$

Readers familiar with the EOPIA technical LTGA specification will notice differences between the risk drivers that are used by EIOPA to produce instantaneous stresses and the risk drivers used in the proxy function described in Section 2.

- » EIOPA yield curve stresses are specified by stressing the full yield curve proportionally or in absolute terms, depending on the current level of a yield curve. Different points on the curve are exposed to different stresses. For the purpose of this analysis we selected the stresses that are applicable to 1-year and 10-year spot rates.

- » EIOPA credit stresses are expressed in terms of percentage losses of a credit-risky portfolio value where the percentage is determined by the credit quality and duration of a portfolio. We have derived the stressed 10-year BBB credit spread so that the corresponding stressed value of a BBB-rated, 10-year credit-risky zero-coupon bond agrees with EIOPA's specified loss for this particular bond.

[6] If NAV(t) is exposed to downward stresses to interest rates, the correlation assumption is 0.5, otherwise 0.

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